Conceptual design of a 20T High Field Dipole Magnet

Author:
Lorenzo Andrea Parrotta

Supervisor:
Dr. Emanuela Barzi

Superconducting R&D
Magnet System Department
Technical Division

Introduction

- Quest for higher fields in accelerator magnets
- New classes of superconducting magnets (HTS)

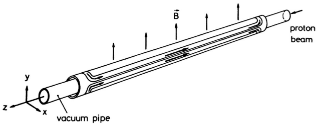
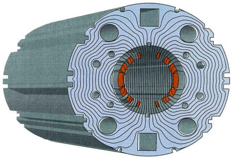


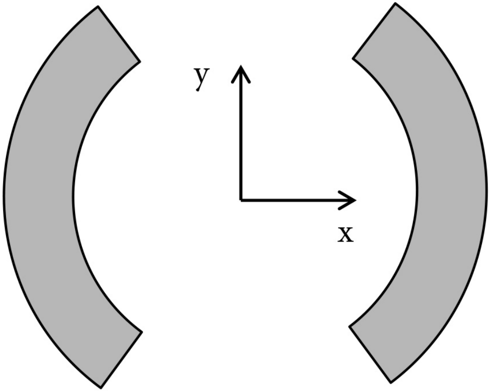
Figure 4.1: Schematic view of a superconducting dipole coil.



Aim of the study

- Investigate the feasibility of a $1\,T$ HTS dipole coil within an existing $11\,T$ dipole
- Design a concept of mechanical structure and a stress management solution for a HTS $5\,T$ insert dipole within a $15\,T$ Nb_3Sn dipole

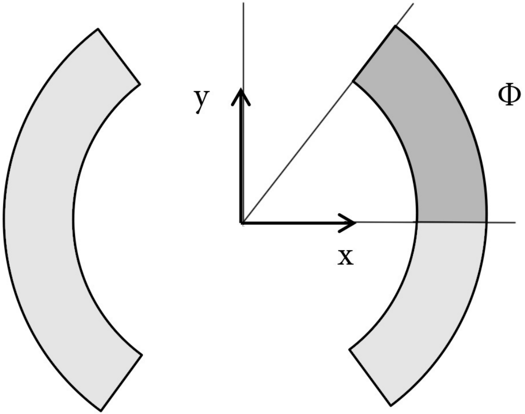
a: internal radius
w: thickness



a: internal radius

w: thickness

ϕ: sector angle



Magnetic Model

Hypotheses:

- current shell distributions
- higher multipole terms neglected
- Yoke effects neglected
- $2D$ model

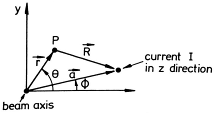
Analitical model of Magnetic Field

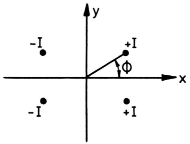
B can be expressed as the curl of the vector potential **A**:

$$\mathbf{B} = \nabla \times \mathbf{A}$$

For 2D problem: $\mathbf{A} = A_z \hat{k}$.

Four line currents with dipole symmetry:





$$\begin{cases} A_z(r,\theta) = \frac{2\mu_0 I}{\pi} \sum_{n=1,3,5..} \frac{1}{n} \left(\frac{a}{r}\right)^n cos(n\theta) cos(n\phi), & r > a \\ A_z(r,\theta) = \frac{2\mu_0 I}{\pi} \sum_{n=1,3,5..} \frac{1}{n} \left(\frac{r}{a}\right)^n cos(n\theta) cos(n\phi), & r < a \end{cases}$$

Magnetic Field contributions

1-Inside the aperture

$$A_z(r,\theta) = \frac{2\mu_0 J_0}{\pi} wr \ cos(\theta) sin(\phi_l)$$

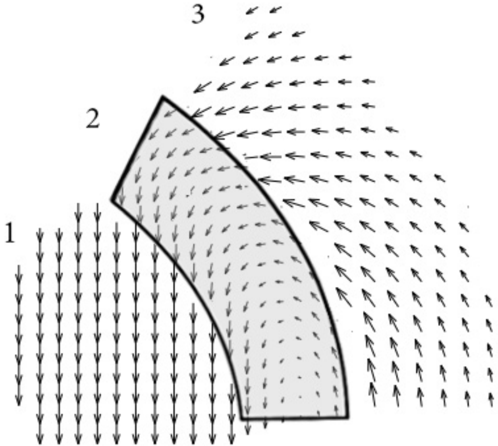
2-On the coil

$$A_z(r,\theta) = \frac{2\mu_0 J_0}{\pi} r \left[(a + w - r) + \frac{r^3 - a^3}{3r^2}\right] cos(\theta) sin(\phi_l)$$

3-On the external region

$$A_z(r,\theta) = \frac{2\mu_0 J_0}{\pi} r \left[\frac{r^3 - a^3}{3r^2}\right] cos(\theta) sin(\phi_l)$$

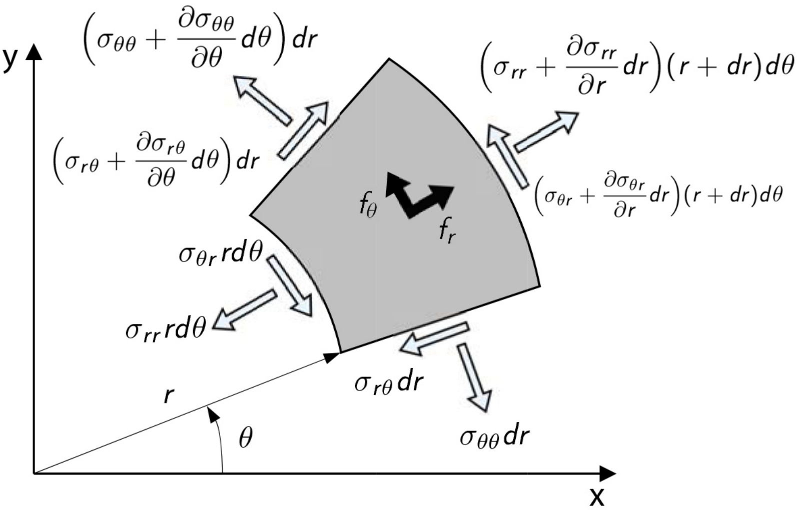
Magnetic Field generated by the coil



Mechanical Model

Hypotheses:

- Linear, Elastic, Omogeneous and Isotropic (IOLE) material
- $2D$ model
- thick membrane sector
- no thermal effects



y
x
r
θ
$\left(\sigma_{\theta\theta} + \frac{\partial \sigma_{\theta\theta}}{\partial \theta} d\theta\right) dr$
$\left(\sigma_{rr} + \frac{\partial \sigma_{rr}}{\partial r} dr\right)(r + dr) d\theta$
$\left(\sigma_{r\theta} + \frac{\partial \sigma_{r\theta}}{\partial \theta} d\theta\right) dr$
$\left(\sigma_{\theta r} + \frac{\partial \sigma_{\theta r}}{\partial r} dr\right)(r + dr) d\theta$
f_θ
f_r
$\sigma_{\theta r} r d\theta$
$\sigma_{rr} r d\theta$
$\sigma_{r\theta} dr$
$\sigma_{\theta\theta} dr$

Two equations from equilibrium along r and θ directions:

$$\begin{cases} \frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + f_r = 0 \\ \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{r\theta}}{\partial r} + 2\frac{\sigma_{r\theta}}{r} + f_\theta = 0 \end{cases}$$

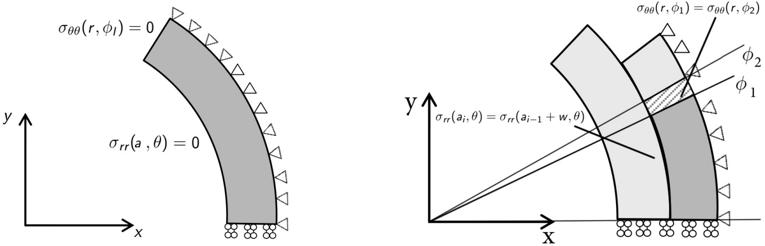
Based on previous studies (Bologna), a generalized plain strain model is considered.

$$\sigma_{zz} = \nu(\sigma_{rr} + \sigma_{\theta\theta}) - \overline{\sigma_{zz}}$$

$$\overline{\sigma_{zz}} = \frac{1}{\pi[(a+w)^2 - a^2]\frac{(\phi_2 - \phi_1)}{2\pi}} \int_{\phi_1}^{\phi_2} \int_a^{a+w} \sigma'_{zz} r \, dr d\theta,$$

being $\overline{\sigma_{zz}}$ and σ'_{zz} the average axial stress and the axial stress for $\epsilon_{zz} = 0$.

Load boundary conditions and constraints



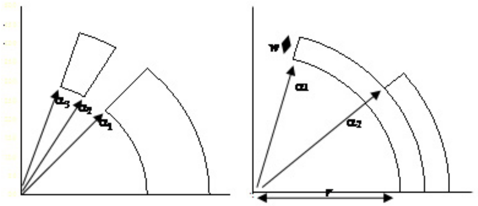
Volume forces (Lorentz's forces)

$$f_r = -B_\theta J_0 = J_0 \frac{\partial(\sum A_{z,i})}{\partial r} \qquad f_\theta = B_r J_0 = J_0 \frac{1}{r} \frac{\partial(\sum A_{z,i})}{\partial \theta}$$

- Shear stress neglected for solving the equations

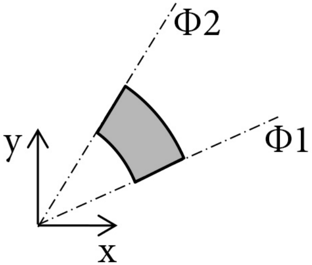
Field Quality requirements

From multipole series:
Skew multipoles $a_n \Longrightarrow$ cancelled by symmetry
Normal multipoles $b_n \Longrightarrow$ can be made to vanish by coil geometry (sector angles and wedges)



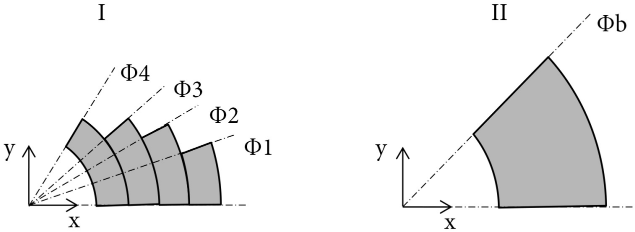
Field Quality optimization

For a coil sector between ϕ_1 and ϕ_2, internal radius a and thickness w:



$$b_n \propto [sin(n\phi_2) - sin(n\phi_1)]\left(\frac{1}{a^{n-2}} - \frac{1}{(a+w)^{n-2}}\right), \quad n = 3, 5, 7, 9, ..$$

An explicative exercise 1

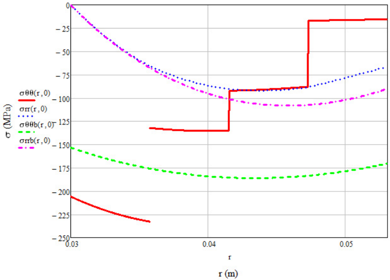


J_0	$800\frac{A}{mm^2}$
a	$30mm$
w_i	$0.19a$
w_b	$4w_i$

ϕ_1	20°
ϕ_2	46°
ϕ_3	57°
ϕ_4	78°

ϕ_b	60°

An explicative exercise 2

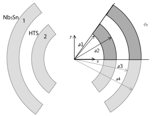


	I	II
b_3	0	0
b_5	0	$-4 \cdot 10^{-3}$
b_7	0	$5 \cdot 10^{-4}$
b_9	0	0
b_{11}	$5 \cdot 10^{-6}$	$-1 \cdot 10^{-5}$

- $\frac{S_I}{S_{II}} = 0.269$
- $\frac{B_I}{B_{II}} = 0.834$

First step: 1 T HTS standalone insert within 11 TNb_3Sn coil

HTS loadline



HTS: BSCCO-2212

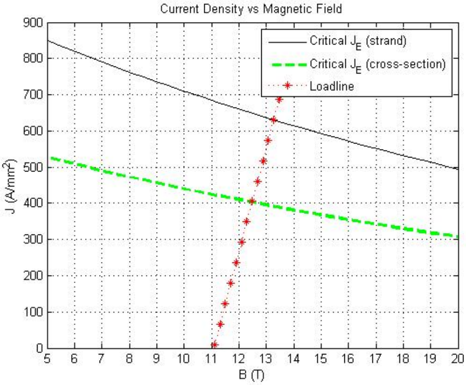
$J_{0,Nb_3Sn} = 800 \frac{A}{mm^2}$

$a_{HTS} = 15mm$

$w_{HTS} = 5mm$

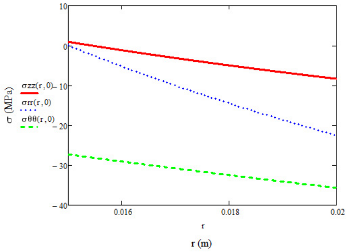
$\phi_l = 60^o$

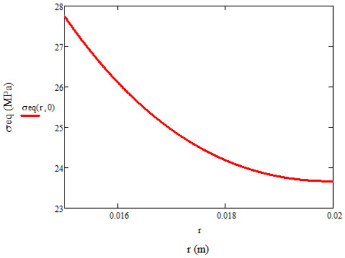
No field quality optimization



First step: $1\,T$ HTS standalone insert within $11\,TNb_3Sn$ coil

$J_{0,HTS} = 300 \frac{A}{mm^2}$
Stress field for $\theta = 0$ (critical section):



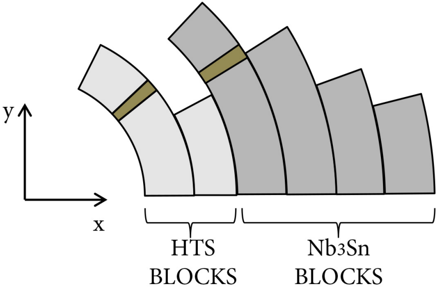


Results

With stresses below the $BSCCO - 2212$ stress limit of about 50 MPa the HTS insert results feasible

Second phase: HTS 5 T insert within a 15 T dipole

- Field quality optimization;
- Material saving;

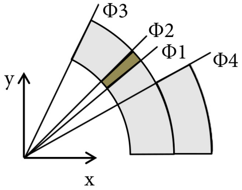


Field quality optimization 1

Separate optimization for HTS blocks and Nb_3Sn blocks

HTS: 2 sectors and 2 blocks in $1st$ sector.
Multipoles made to vanish until b_9.

$$\begin{cases} [sin(3\phi_1) - sin(3\phi_2) + sin(3\phi_3)]\left(\frac{1}{a} - \frac{1}{a+w}\right) + sin(3\phi_4)\left(\frac{1}{a+w} - \frac{1}{a+2w}\right) = 0 \\ [sin(5\phi_1) - sin(5\phi_2) + sin(5\phi_3)]\left(\frac{1}{a^3} - \frac{1}{(a+w)^3}\right) + sin(5\phi_4)\left(\frac{1}{(a+w)^3} - \frac{1}{(a+2w)^3}\right) = 0 \\ [sin(7\phi_1) - sin(7\phi_2) + sin(7\phi_3)]\left(\frac{1}{a^5} - \frac{1}{(a+w)^5}\right) + sin(7\phi_4)\left(\frac{1}{(a+w)^5} - \frac{1}{(a+2w)^5}\right) = 0 \\ [sin(9\phi_1) - sin(9\phi_2) + sin(9\phi_3)]\left(\frac{1}{a^7} - \frac{1}{(a+w)^7}\right) + sin(9\phi_4)\left(\frac{1}{(a+w)^7} - \frac{1}{(a+2w)^7}\right) = 0 \end{cases}$$



Field quality optimization 2

- $J_{0,HTS}$ and a are chosen;
- 5 unknowns: $\phi_1, \phi_2, \phi_3, \phi_4, w$;
- Another condition: $B_{0,bore} = 5\,T$.

A numerical solution was found using *MATLAB* solver.

Field quality optimization 2

- $J_{0,HTS}$ and a are chosen;
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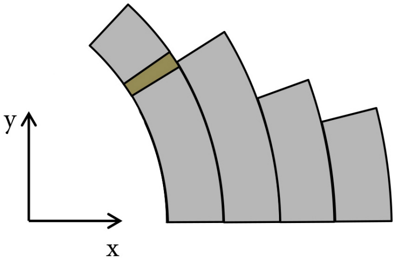
Field quality optimization 2

- $J_{0,HTS}$ and a are chosen;
- 5 unknowns: $\phi_1, \phi_2, \phi_3, \phi_4, w$;
- Another condition: $B_{0,bore} = 5\,T$.

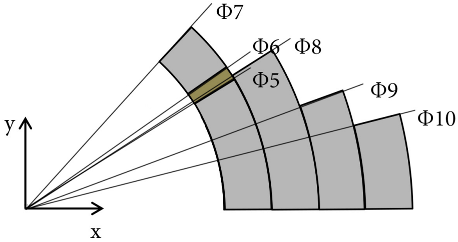
A numerical solution was found using *MATLAB* solver.

Field quality optimization 3

Nb_3Sn: 4 sectors and 2 blocks in 1*st* sector.
Results from HTS optimization used as input for Nb_3Sn optimization:
$a_{Nb_3Sn} = a_{HTS} + 2w_{HTS}$.

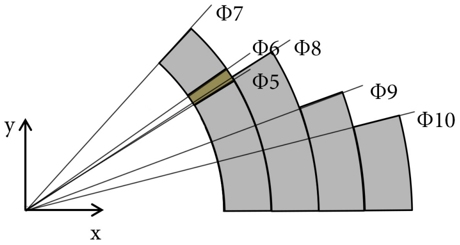


Field quality optimization 4



- $J_{0,HTS}$ is chosen and a is given by HTS optimization ;
- 7 unknowns: $\phi_5, ..., \phi_{10}, w$;
- 5 conditions: $b_3 = b_5 = b_7 = b_9 = 0$; $B_{0,bore} = 15\,T$.
- 2 parameters

Field quality optimization 4



- $J_{0,HTS}$ is chosen and a is given by HTS optimization ;
- 7 unknowns: $\phi_5, \ldots, \phi_{10}, w$;
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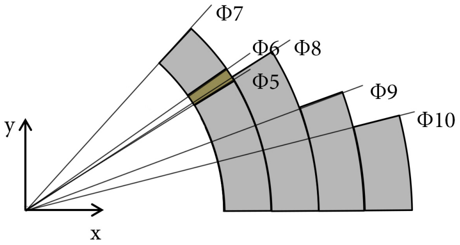
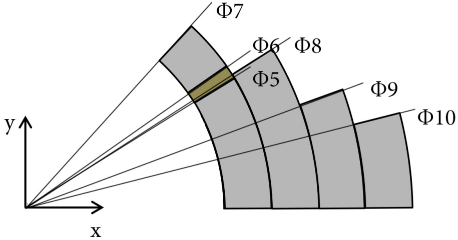
Field quality optimization 4



- $J_{0,HTS}$ is chosen and a is given by HTS optimization ;
- 7 unknowns: $\phi_5, ..., \phi_{10}, w$;
- 5 conditions: $b_3 = b_5 = b_7 = b_9 = 0$; $B_{0,bore} = 15\,T$.
- 2 parameters

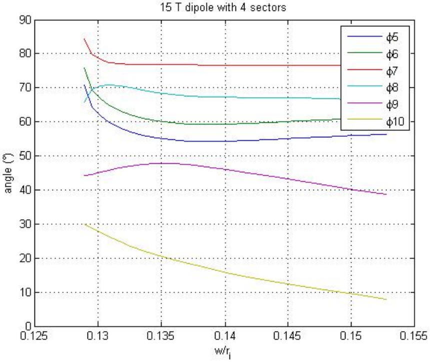
Field quality optimization 4



- $J_{0,HTS}$ is chosen and a is given by HTS optimization ;
- 7 unknowns: $\phi_5, \ldots, \phi_{10}, w$;
- 5 conditions: $b_3 = b_5 = b_7 = b_9 = 0$; $B_{0,bore} = 15\,T$.
- 2 parameters

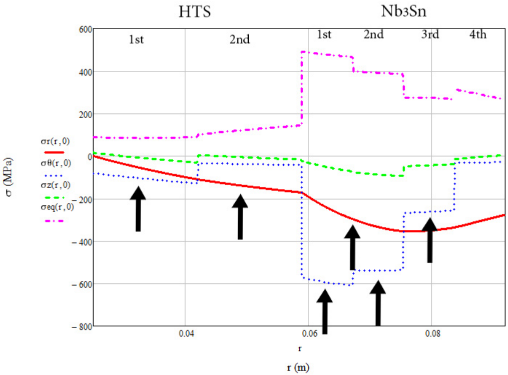
Field quality optimization 5

e.g. chosen a 5° wedge:



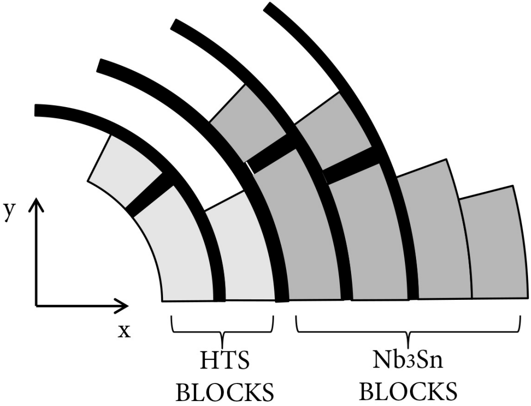
Stress Field

$J_{0,HTS}$	$250 \frac{A}{mm^2}$
J_{0,Nb_3Sn}	$800 \frac{A}{mm^2}$
a_{HTS}	$25mm$
w_{HTS}	$0.678 a_{HTS}$
w_{Nb_3Sn}	$0.140 a_{Nb_3Sn}$



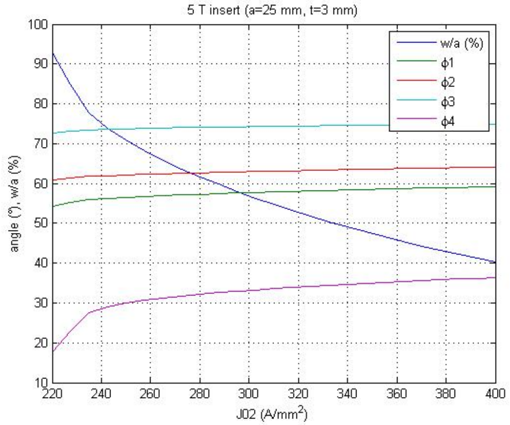
Proposed structure - Slotted shells

Presence of ***structural wedges***.



Structure optimization 1

HTS blocks



Structure optimization 2

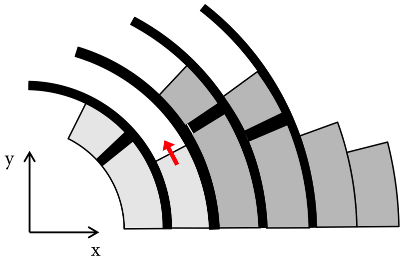
Nb_3Sn blocks

- At least split 2*nd* sector in two blocks in order to reduce azimuthal stress
- Optimization system to be solved for the proposed structure

Structure optimization 3

Crossed optimization

Adding material in the regions of low stress levels and compensate the multipoles arisen.



$$b_{9,HTS} \neq 0 \Longrightarrow b_{9,Nb_3Sn} = -b_{9,HTS}$$

Next steps

- FEM simulation of the entire structure
- Need for $BSCCO - 2212$ material characterization
- Look at $YBCO$ inserts